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ABSTRACT

The existence of probability misconceptions at all levels has been well documented. Furthermore, these misconceptions have been shown to be widespread and highly resistant to change. Previous research has shown considerable success in overcoming misconceptions in the short term by basing the knowledge reconstruction process on problems which draw out beliefs held by students which are in agreement with accepted theory and which are therefore expected to receive correct responses. Such problems are referred to as anchoring situations or anchors. Anchoring probability situations which are conceptually analogous to misconception-prone/target probability situations were generated and tested with secondary mathematics students. The testing showed that probability misconceptions were common but also that anchors for overcoming these misconceptions could be generated. Anchoring situations were effectively used in overcoming students' probability misconceptions in the short term. A follow-up study showed that short term effects were retained over a 6-month period, thereby establishing the long term effectiveness of the approach. (Contains 35 references and 2 tables of data. Appendixes present a sample interview protocol and definitions of interview ratings.) (Author/RS)

Using Analogies to Produce Long Term Conceptual Change: Overcoming High School Mathematics Students' Probability Misconceptions

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Abstract

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Anchoring probability situations which are conceptually analogous to misconception-prone/target probability situations were generated and tested with secondary mathematics students. The testing showed that probability misconceptions were common but also that anchors for overcoming these misconceptions could be generated. Anchoring situations were effectively utilized in overcoming students' probability misconceptions in the short term. A follow-up study showed that short term effects were retained at the rate of 0.65 over a six month period thereby establishing the long term effectiveness of the approach.

Introduction

In the last two decades, there have been urgent calls from various educational organizations such as the National Council of Teachers of Mathematics for curriculum change in mathematics. In their considered opinion, students are not being adequately prepared for the challenges of the next century. One of the recommendations which has been put forth by these educational bodies as well as by prominent individual researchers is that students need greater exposure to topics which are more relevant to the needs of everyday situations which would be encountered by a majority of students in the future. One of these topics is probability (Carl, 1989; National Research Council, 1989; Romberg, 1992). Indeed, it is a singularly unusual day in which one does not encounter some reference to probability such as "The probability that it will rain today is 20%" or "Joe hasn't had a hit in his last six times at bat so he's due for a hit".

Although the study of probability is highly relevant for understanding numerous everyday situations, it is also one of the topics in mathematics which is most prone to misconceptions (Shaughnessy, 1981; Hope & Kelly, 1983; Jacobsen, 1989). Consequently, it becomes necessary to identify and understand these misconceptions and how they can be overcome.

The Purpose of the Research

The purpose of the research was to investigate secondary mathematics students' conceptual understanding of common, everyday situations involving probability for the ultimate purpose of overcoming the misconceptions which exist in this area. In particular, the research was concerned with establishing the long term effects of the analogies approach which had previously been shown to be effective in producing conceptual change but which had not established the permanence of the change (Fast, 1994).

The Significance of the Study

Garfield and Ahlgren (1988) state that ".... little seems to be known about how to teach probability and statistics effectively" (p. 45) and yet the study of this topic is of vital importance. A substantial portion of the difficulty in teaching the topic of probability can be attributed to the misconceptions which permeate students' thinking in this area. These have been well documented in previous research as has their resistance to change. Attempts to overcome various mathematics misconceptions in general, and probability misconceptions in particular, have met with limited success. This researcher, however, has documented remarkable results in overcoming probability misconceptions through the use of anchors and analogical reasoning (Fast, 1994).

The present study is highly significant in that it provided evidence of the long term effectiveness of using analogies in overcoming probability misconceptions which previously had only been shown to be effective in the short term. Overcoming misconceptions is an essential component of acquiring a mathematically-correct understanding of probability situations as they are studied in the classroom and as they occur in everyday situations.

Theoretical Background

Much of the original research on probability misconceptions was done by Kahneman and Tversky. They showed that these misconceptions are prevalent even among college students who have a statistics background. Two of the most common types of misconceptions are the result of relying on the representativeness heuristic (Kahneman & Tversky, 1972; Tversky & Kahneman, 1971, 1977) and the availability heuristic (Tversky & Kahneman, 1973, 1977). These heuristics are constructed by individuals as a general approach for responding to various probability situations. The representativeness heuristic, however, misleads the student to believe that even a small sample should be representative of the population from which it is taken. It is thus not appropriate to apply these heuristics in every situation. For example, if a coin is tossed a number of times and the result is a series of "heads", it is commonly believed that the probability of obtaining "tails" on a subsequent toss is greater than obtaining "heads" so that the sample will represent the outcomes of the theoretical population. This type of belief has also become known as the "gamblers' fallacy".

The representativeness heuristic manifests itself in various other ways as well such as believing that in a family with five children, the birth order BGGBG is more common than BBBBB since BGGBG is more representative of the type of sample one normally observes which contains a mixture of boys and girls as compared to a family with children of only one sex. In this example, the individual may also be utilizing the availability heuristic. In attempting to answer the question, the individual recalls various examples of the required situation which are available to him/her, and it is likely that most of these examples of families with five children will include children of both genders.

The representativeness heuristic also manifests itself in lottery situations where its use may result in the belief that a choice of numbers such as 3 8 19 27 32 41, in the 6/49 lottery (correctly choosing 6 numbers from 1 to 49 - order not important), will have a better chance of winning than 1 2 3 4 5 6 since the former is more representative of the results which usually occur.

Unfortunately, the misconceptions once formed are highly resistant to change (Shaughnessy, 1985; Konold, 1988). It appears that one of the reasons for this is that concept construction requires considerable effort and consequently there exists a great reluctance to abandon these concepts in the face of contradictory evidence (Brown & Clement, 1987).

Overcoming Misconceptions and Constructivist Theory

Constructivist theory tells us that students come to understand relationally through their own construction of knowledge (Glaser, 1991). This implies that they have constructed concepts or schemata for learned situations which allows answering 'why' questions and making problem solving adaptations to related tasks. Attempts to "pour" knowledge into someone's mind usually results only in instrumental understanding which suggests the student only knows <u>how</u> to do a particular task (Skemp, 1989). It seems reasonable therefore to also apply the constructivist approach to the concept reconstruction process which is necessary for overcoming misconceptions. Since all knowledge construction must begin with what the student already knows, the logical approach is to generate situations to which the student will respond correctly, so that an anchor can be established on which the construction process can begin. Eventually it should be possible to establish a sequence of anchors which assist in the construction process of building a bridge from what the student knows or believes to be correct to what the student has misconceived.

The use of anchors or supports in knowledge construction has been advocated by various educators and researchers including Glaser (1991) and Gorsky & Finegold (1992). They state that in accordance with constructivist theory, when new topics are introduced, teachers should initially provide a framework or scaffolding on which the learners will be able to elaborate their versions of a basic knowledge structure.

In summary, the present research is based on six theoretical assumptions:

1) probability misconceptions are prevalent in students;
2) students learn by constructing their own knowledge which often contains conflicting schemata - some which are mathematically-correct and others which are mathematically-incorrect;
3) constructed knowledge, whether it is mathematically-correct or incorrect, is highly resistant to change;
4) knowledge construction must take into consideration an individual's prior knowledge and it is most effective to begin the construction or reconstruction process with the mathematically-correct schemata the individual already possesses;
5) the specific representation of a problem is a key determinant in the student's ability to solve the problem - some representations activate schemata which are appropriate for solving the problem whereas other representations activate schemata which are based on misconceptions;
6) students can be assisted in constructivist learning, particularly in overcoming misconceptions, through supportive frameworks such as a series of anchoring situations.

The Objectives

1. To utilize the anchoring situations which were generated to produce conceptual change resulting ultimately in overcoming the probability misconceptions revealed in the target situations.
2. To determine the permanence of the conceptual change which had been affected as a result of the analogies approach.

The Methodology

Developing the Instrument

During the development of the instrument administered in this study, a variety of situations similar to the ones extensively quoted in the literature as misconception-prone probability situations were matched with researcher-generated anchoring situations conceptually isomorphic to the misconception-prone situations. It was hypothesized that the anchoring situations were more likely to activate mathematically-correct schemata than the misconception-prone situations due to the various techniques which were utilized in generating the anchoring situations. These techniques included presenting the problem from a different perspective; utilizing concrete or familiar situations; changing the numerical quantities in order to present extreme cases, etc.

The instrument, in its various developmental stages, was subjected to pilot-testing with 24 students of varying probability backgrounds and educational levels from senior high school to graduate level. Six different editions of the instrument were developed and tested with the goal of increasing the effectiveness of each subsequent edition in revealing misconceptions in *Version A* and producing anchors in *Version B*. As a result of this testing, a seventh edition was prepared in which ten misconception-prone situations were compiled as *Version A7* of the *WDYTTCA* (*What Do You Think The Chances Are?*) instrument. Their ten analogous counterparts were placed in *Version B7* of the *WDYTTCA* instrument. Questions regarding the likelihoods of events were posed in a multiple-choice format. Justifications for the likelihoods were requested in constructed-response format. Confidence lines were included so that the respondent could indicate his/her degree of confidence in the responses to the multiple-choice questions on a continuum from 0 to 3 with demarcations 0 (just a guess); 1 (not very confident); 2 (fairly confident); and 3 (I'm sure I'm right).

An example of a question in *WDYTTCA Version A7* and its analogous counterpart in *Version B7*, with reasons for its inclusion, is given below.

Question 3 *(Version A7)*

Your sports team finishes first in its league at the end of the season and so you consider it is the best team. However you must compete in a playoff series against the second place team in the league to determine the champion. Would a 5 game series or a 9 game series give you a better chance of winning the championship, or doesn't it make any difference?

a) a 5 game series gives you a better chance;

b) a 9 game series gives you a better chance;

c) it makes no difference.

Those who rely on the representativeness heuristic believe that both the larger sample 9 game series and the smaller sample 5 game series will reflect the population (i.e. that your team is better) equally well in all cases. They will probably respond that it makes no difference, not taking into consideration that the larger sample, the 9 game series, would be more likely to show which team is actually better.

Question 3 *(Version B7)*

Your sports team finishes first in its league at the end of the season and so you consider it is the best team. However you must compete in a playoff series against the second place team in the league to determine the champion. Would a sudden-death 1 game playoff or a 5 game series give you a better chance of winning the championship, or doesn't it make any difference?

a) a sudden-death 1 game playoff gives you a better chance;

b) a 5 game series gives you a better chance;

c) it makes no difference.

Experience tells you that you do not win every game even if you are the best team and therefore a sudden-death 1 game playoff is very risky whereas a 5 game series gives you a better chance to show your talent. It is the extremity of the numerical quantity "1" in the 1 game series which makes the correct choice more obvious. Consequently, this situation provides an anchor for the statistically-correct concept that a larger sample is more likely to reflect the characteristics of the population from which it is taken than a smaller sample.

Administering WDYTTCA Version A7 and Version B7

The seventh edition of the *WDYTTCA* instrument was tested with a group of 41 secondary mathematics students. Seventeen of these students were interviewed. The participants began by first completing *Version A7* of the *WDYTTCA* instrument. When they were finished they immediately went on to *Version B7*. They were instructed not to go back to *Version A7* after beginning *Version B7*. The objective was to determine the effectiveness of the situations in *Version B7* in eliciting correct responses and appropriate justifications especially in regard to those questions whose analogous counterparts in *Version A7* had received incorrect responses and misconception-revealing justifications.

Volunteers were interviewed individually within a week of completing the written instrument. The purpose of the interview was first to establish whether or not the possible misconceptions which had been revealed in the *WDYTTCA* instrument were true misconceptions. Second, and most important, the interview attempted to determine the effectiveness of using analogies to affect conceptual change regarding the probability misconceptions which had been revealed. The participants were presented with the situations in *Version A7* of *WDYTTCA* to which they had responded incorrectly (possible misconceptions) when at the same time they had responded correctly and with confidence (anchors) to the analogous counterparts in *Version B7* . At this point, some students were already prepared to change some of their mathematically-incorrect answers in *Version A7*.. They indicated that the *Version B* situations had made them realize the error of their thinking in the *Version A7* counterpart. These were viewed as particularly successful results in that the instrument itself had prompted the conceptual changes. In most cases however, it was necessary for the interviewer to engage the students in a process of analogical reasoning in an attempt to guide the knowledge reconstruction process which hopefully would ultimately result in the participant changing incorrect responses to correct responses in the *Version A7* situations (See Appendix A for Interview Protocol). The change from an incorrect to a correct response combined with an indication of being at least "fairly confident" in the correct response was deemed as evidence of the conceptual change necessary for overcoming the misconception in the long term. The success rate for this conceptual change was found to be 0.72 (See Appendix B for Definitions of Interview Ratings).

The Long Term Study

The long term study began with two grade ten classes comprising 47 students in total and one grade eleven class of 25 students. They were given *Version A7* and *Version B7* of the *WDYTTCA* instrument. Twenty-four of these students (15 from the grade ten class and 9 from the grade eleven class), who agreed to participate in the study on a long term basis, were interviewed. All procedures were as in the previous study. Six months later, 20 of these 24 students, who had now advanced one grade, were given *Version A8* of the *WDYTTCA* instrument. The remaining four were absent due to illness. The questions on *Version A8* were designed to be analogous to those in *Version A7* and as likely to elicit misconception-revealing responses.

Findings of the Long Term Study

The results in *Version A8* , given six months after the original testing and interviewing, were compared to the results in *Version A7* to determine the long term effectiveness of using analogies to overcome probability misconceptions. Sixty-five interviewed situations were available for comparison. Forty-nine of these situations had resulted conceptual changes indicative of misconceptions being overcome through the use of analogies and the interview process. Thirty-two of these 49 resulted in correct responses on *Version A8* given six months later which translated to a 65% long term retention rate (See Table I). Seventeen of the 49 situations were not successful in the long term and in those situations students' responses on *Version A8* indicated that they had reverted to thinking indicative of misconceptions. As might be expected, most interviewed situations which were not successful in producing conceptual change were again answered incorrectly in the final version. There were eight such situations out of the 65 interviewed situations retested. Only one situation actually received a correct response on *Version A8* when the interview had not been successful.

Table I Comparison of *Version A8* Responses to Interview Results

	Correct Responses - A8	*Incorrect Responses - A8*
Interview Results		
Successful	32	17
Unsuccessful	8	1
Partially Successful	3	4

Interviewed Situations Retested with *Version A8* N=65

For further analysis, the results of non-interviewed situations could also be considered. These serve as a control, a basis of comparison for the interviewed situations retested. One hundred thirty-five situations, which had not received any intervention in terms of an interview, were retested on *Version A8* after a six month interval. Thirty-seven did not produce a correct result on *Version A8* when at the same time an incorrect answer had been recorded on *Version A7*. Forty-nine of the 135 produced a correct result on *Version A8* when a correct result had also been recorded initially on *Version A7*. These two results are consistent with expectations and show that in 86 out of 135 situations or 63.7% of the time no change occurred when no interview occurred (See Table II).

Table II Comparison of *Version A8* Responses to *Version A7* Responses When Conceptual Change was Not Attempted with an Interview

	A8 Responses - Correct	*A8 Responses - Incorrect*
A7 Responses		
Correct	49	19
Incorrect	30	37

Non-Interviewed Situations Retested with *Version A8* N=135

It should be noted that 16 of the 30 situations that were answered incorrectly on *Version A7* but correctly on *Version A8* were answered correctly on *Version B7*. Thus the analogy in *Version B7* may in itself have resulted in a conceptual change resulting in a change in answers from *Version A7* to *Version A8* even though no interview took place.

Thirty of the 135 non-interviewed situations received a correct response on *Version A8* when *Version A7* had received an incorrect response. This reliability concern is not as great as it might appear for a closer examination revealed that 16 of these situations did receive a correct answer on *Version B7*. *Version B7* provided situations analogous to *Version A7* but presented in such a way as to hopefully elicit thinking in accordance with accepted mathematical theory and thus produce a correct answer. It may therefore have been the case that in some of those situations, the *Version B7* analogy was sufficient in itself in producing a long term conceptual change. This however needs further investigation.

Finally, 19 of the 135 situations not involved in interviews were answered correctly on *Version A7* but not on *Version A8*. Some variability in responses to similar situations without significant intervention is characteristic of students when misconceptions exist. The students may know the accepted mathematical concept but they do not really believe it so at one time the response reflects the accepted mathematical knowledge and on another occasion the actual belief. This characteristic of students with misconceptions creates reliability issues which are best resolved through extensive interviews which attempt to determine the deep knowledge or more precisely the deep beliefs of the students.

Conclusions

The research described in this paper attempted to utilize analogies to produce conceptual change in students' probability misconceptions and to determine the long term effects of the conceptual change. The results of a previous study had shown that an analogies approach could result in considerable conceptual change particularly in view of the strong resistance to change noted in previous research on misconceptions. Although conceptual change could be produced using analogies, that study did not investigate the permanence of the conceptual change. The present study therefore attempted to answer the question of long term retention.

The results of the present study showed a conceptual retention rate of 0.65 as a result of the intervention using analogies. The intervention time for each student and each situation was brief, no follow-up intervention was utilized, and a fairly lengthy time period occurred between the original testing and the final testing. In view of these considerations, the long term effects of using analogies to produce conceptual change and overcome high school students' probability misconceptions could be described as quite successful.

Analogies are alternate representations of a situation. Students are familiar with using alternative representations such as diagrams, tables, and charts in clarifying situations and solving problems but using analogies may be unfamiliar to most therefore this requires introduction and practice. No one analogy will necessarily be effective for all individuals therefore for this approach to be successful, the teacher should have a variety of analogies prepared to be introduced as the need arises. Ultimately, students should be motivated to construct their own analogies which make situations meaningful and understandable to them. The broader aspect of the research involves generating or finding analogies to facilitate students' understanding in all areas of mathematics as well as other disciplines.

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Appendix A: Question 8 and Sample Interview Protocol

Version A7
8. Suppose John Olerud has a batting average of .333 (1 hit in 3 times at bat) against Jimmy Key. In a certain game he comes up to bat 6 times against Key. He has no hits the first 3 times at bat. What is your best guess as to how many hits he will get in his last 3 times at bat?

a) 0 b) 1 c) 2 d) 3

Version B7
8. Suppose Joe Carter has a batting average of .250 (1 hit in 4 times at bat) against Jimmy Key. In a game in June, Carter gets no hits out of 4 times at bat against Key. Two months later in August, Carter again comes to bat 4 times against Key. How many hits do you expect him to get this time?

a) 0 b) 1 c) 2 d) 3 e) 4

I: Let's have another look at one of the questions you answered on *WDYTTCA*. Here is Question 8 in *Version A*. Would you read the question again and look at the answer you gave. [Student reads the question.]

I: So you said that Olerud should get 2 hits in the last 3 times at bat to maintain his average of .333. Do you still agree with this answer? [If the student says "no", then the interviewer will ask why not and what her new answer is and why. This is to determine if the student has been influenced by *Version B* of *WDYTTCA* or any other factor since answering the question. If the student already knows the correct answer and can give an appropriate explanation, then it can be assumed that the student has already overcome the misconception. If the student says she agrees with her original answer, she will then be presented with the *Version B* situation.]

I: Now let's have a look at Question 8 in *Version B*. Would you please read the question again and look at the answer you gave. [Student reads the question.] You said that Carter would get 1 hit against Key. Do you still agree with your answer? [If the student says "no" and gives the wrong answer with an explanation indicating a misconception as in *Version A*, then the situation has lost its role as an anchor. The student will then be asked why she changed her mind. If the student, however still agrees with her former correct response, she will then be asked ......]

I: Look at this Question 8 on *Version B* and this Question 8 on *Version A*. Do you see a similarity between the two questions? [If the student says no, similarities can be pointed out, but if the student says yes as expected .....]

I: Okay, what similarities do you see? [Similarities are established and then ....]

I: What about your answers for the two questions, are you using the same reasoning to answer both? [If she answers yes, further probing can reveal the difference. If she answers no ......]

I: So why did you use different reasoning to answer the two questions? [The difference between the two questions is simply a time difference between the batting sessions so it is expected that the student will point out this difference. Then .....]

I: Suppose Carter comes to bat 4 times against Key 1 month later instead of 2 months later, how many hits do you expect him to get? [The idea is to gradually decrease the time interval between the two batting sessions until they occur in the same game as in *Version A* while at the same time having the student maintain her anchoring response in *Version B*. If the anchor is not a brittle anchor then this can be done.]

I: So, how many hits do you think Olerud would get in the last 3 times at bat? [If the student now changes her answer to "1" in *Version A* and gives an appropriate explanation as she did in *Version B*, she will then be asked how confident she is in her new answer. She will be presented with the confidence line. If she indicates at least a 2 (fairly confident) the result will be recorded as a "Success" in overcoming her misconception.]

Appendix B: Definitions of Interview Ratings

Successful - when a participant changes his/her incorrect response to a correct response in *WDYTTCA Version A7* using the analogy in *Version B7* and/or an alternate analogy provided by the interviewer and indicates that he/she is at least fairly confident (level 2 or higher on the confidence scale) in the revised response. This result was assigned 1 point.

Partially Successful - when a participant changes his/her incorrect response to a correct response in *WDYTTCA Version A7* but indicates that he/she is not very confident (level 1 or lower on the confidence scale) in the revised response or it appears that the revised response is probably partly due to aspects other than an analogous situation. This result was assigned 0.5 points.

Unsuccessful - when a participant does not change his/her incorrect response to a correct response in *WDYTTCA Version A7* . This result was assigned 0 points.

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